Learning objectives

- Learn about angle bisectors in triangles, their properties and how to use them to solve problems.

Review: Midesgments; perpendicular bisectors and angle bisectors.

Midsegments join two midpoints and are parallel to the third side and half the length of the third side.

Perpendicular bisectors connect a midpoint with the opposite vertex and bisect both the line segment and the vertex. Every point on the perpendicular bisector is equidistant from the endpoints of the line segment.

Angle bisectors cut an angle in half and are equidistant from the sides of the angle.

Discovery learning!

Draw a circle centered at point ***C***, then pick three points on the circle and connect them (making a triangle).

Draw three lines from the center of the circle perpendicular to each side of the triangle.

What conjectures (educated guesses) can you make about the how the sides of the triangle are divided by those lines?

Concurrency in triangles

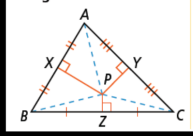
When three or more lines intersect at the same point, they are ***concurrent***. The point at which they intersect is called the ***point of concurrency***.

In this case that point is ***C***.

For any triangle, certain sets of lines are always concurrent. Two of these sets of lines are the ***perpendicular bisectors*** of the triangle's three sides and the ***bisectors*** of the triangle's three angles.

Theorem 5-6: Concurrency of Perpendicular Bisectors Theorem

Diagram



Symbols

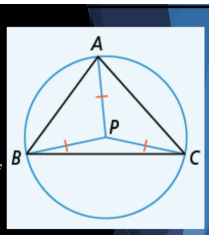
Perpendicular bisectors $\overline{PX}$, $\overline{PY}$, and $\overline{PZ}$ are concurrent at P.

$PA = PB = PC$

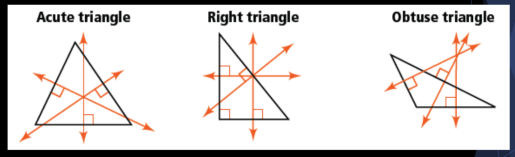
The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

The point of concurrency of the perpendicular bisectors of a triangle is called the **circumcenter of the triangle.**

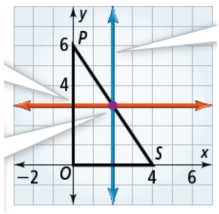
Since the circumcenter is equidistant from the vertices, you can use the circumcenter as the center of the circle that contains each vertex of the triangle. The circle is thus **circumscribed about** the triangle.



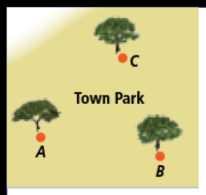
The *circumcenter* of a triangle an be **inside**, **on** or **outside** the triangle.



What are the coordinates of the circumcenter of the triangle with vertices $P(0, 6)$, $O(0, 0)$, and $S(4, 0)$? Hint: it's a right triangle, so where should the center be?

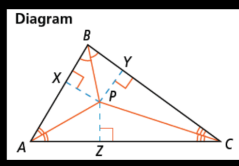


If I want to place a bench equidistant from three trees in a park, where should I put it?



Theorem 5-7: Concurrency of Angle Bisectors Theorem

Diagram



Symbols

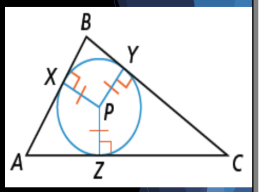
Angle bisectors $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are concurrent at P.

$PX = PY = PZ$

The bisectors of the angles of a triangle are concurrent at a point *equidistant* from the sides of the triangle.

The point of concurrency of the angle bisectors of a triangle is called the ***incenter* of the triangle.**

For any triangle, the *incenter* is always inside the triangle. In the diagram, points *X*, *Y*, and *Z* are *equidistant* from *P*, the incenter of *ABC*. *P* is the center of the circle that is **inscribed inside** the triangle.



$GE = 2x - 7$ and $GF = x + 4$. What is GD?